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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



## THESIS

### MEASURING SURFACE COMBATANT FLEET EFFECTIVENESS

by

Michael A. Crary

September 1999

Thesis Advisor:

Linda K. Nozick

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**MEASURING SURFACE COMBATANT FLEET EFFECTIVENESS**

Michael A. Crary  
Lieutenant, United States Navy  
B.S., United States Naval Academy, 1992

Submitted in partial fulfillment of the  
requirements for the degree of

**MASTER OF SCIENCE IN OPERATIONS RESEARCH**

from the

**NAVAL POSTGRADUATE SCHOOL**  
**September 1999**



## ABSTRACT

How is the effectiveness of surface combatant ships in a Major Theater War measured? While Measures of Effectiveness for an individual ship can include its number of missiles, speed, and endurance, it is difficult to find a Measure of Effectiveness credible to experienced warplanners for a fleet of ships.

This thesis develops a Fleet Measure of Effectiveness (FMOE) to forecast the success of surface combatants in a Major Theater War. We define FMOE, discuss the elements that contribute to its calculation, and justify why a distribution for FMOE is preferable to a point estimate. This thesis also shows how to integrate samples from a distribution of inputs and human judgment into an optimization model. Finally, FMOE is implemented through case studies that examine the impact logistics support has on fleet effectiveness and show how FMOE distributions can be used to compare the effectiveness of various surface combatant fleets.



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## EXECUTIVE SUMMARY

How is the effectiveness of surface combatant ships in a Major Theater War measured? While Measures of Effectiveness for an individual ship can include its number of missiles, speed, and endurance, it is difficult to find a Measure of Effectiveness credible to experienced warplanners for a fleet of ships.

The performance of Navy ships in a major war is a critical issue. United States strategic policy is based on its ability to fight and win two nearly simultaneous Major Theater Wars. Force requirements for the Navy are determined by the estimated mix of ships necessary to win these wars. One surface combatant costs in excess of 500 million dollars: with average ship lifespans in excess of 30 years, a flawed ship mix can be costly and difficult to correct. A credible way to forecast the performance of ships in a Major Theater War would result in a more robust fleet and could save billions of dollars.

The Fleet Effectiveness Model (FEFM) provides decision-makers with a tool to measure the performance of surface combatants in a Major Theater War. FEFM not only measures the performance of a fleet of ships but also illustrates how surface combatant mission capabilities affect fleet performance. It also verifies the strong relationship between logistics support and fleet effectiveness. Most importantly, it allows planners to address the many “what if” scenarios concerning fleet composition.

Acquiring and designing ships will always be an expensive process. FEFM demonstrates that mathematical programming can help measure fleet effectiveness and therefore aid in the design of a more effective fleet.



## I. INTRODUCTION

How is the effectiveness of surface combatant ships in a Major Theater War (MTW) measured? While Measures of Effectiveness (MOE) for an individual ship can include its number of missiles, speed, and endurance, it is difficult to find a Measure of Effectiveness credible to experienced warplanners for a fleet of ships.

The performance of Navy ships in a major war is a critical issue. United States strategic policy is based on its ability to fight and win two nearly simultaneous Major Theater Wars. Force requirements for the Navy are determined by the estimated mix of ships necessary to win these wars. One surface combatant costs in excess of 500 million dollars: with average ship lifespans in excess of 30 years, a flawed ship mix can be costly and difficult to overcome. An accurate way to forecast the performance of ships in a Major Theater War would result in a more robust fleet and could save billions of dollars. Figure 1 illustrates the Fleet Effectiveness Problem.

This thesis develops a Fleet Measure of Effectiveness (FMOE) to forecast the success of surface combatants in a Major Theater War (MTW). FMOE can then be used to compare the capabilities of various fleets of ships. It also is used to examine a number of issues including the impact logistics has on fleet effectiveness. Before describing FMOE, it is necessary to define the elements of a MTW. The remainder of this chapter describes the Major Theater War scenario for the Far East in 2015 based on the results of a study conducted by the Applied Physics Laboratory (APL) at The Johns Hopkins University.

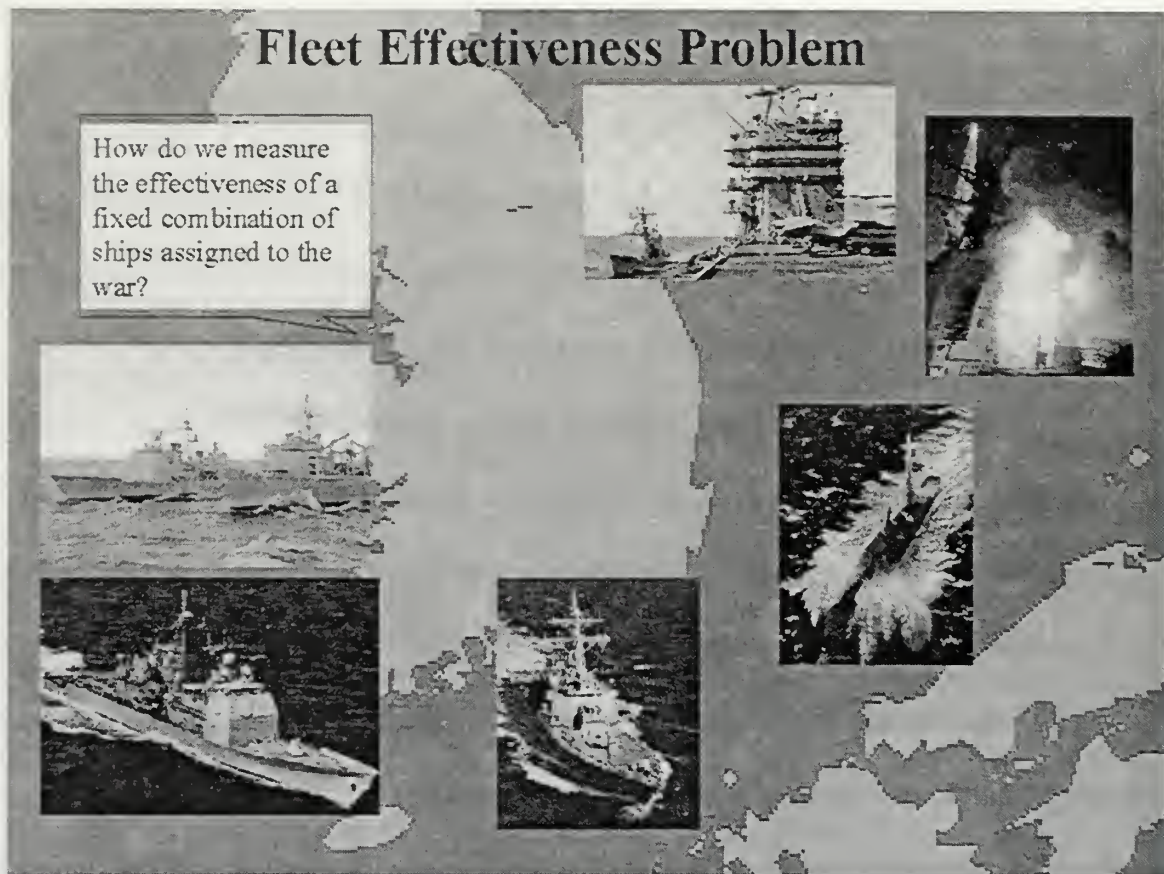


**Figure 1.** The Fleet Effectiveness Problem. We seek a method to measure the effectiveness of ships participating in a Major Theater War (MTW) in the Far East. The question is: how many ships of what classes are needed to collectively win the war?

## **A. PRECEDING WORK**

### **1. Surface Combatant Force Level Study Discussion**

In 1998 the Applied Physics Laboratory (APL) at The Johns Hopkins University was tasked by the Director, Surface Warfare Division (N-86) to conduct a Surface Combatant Force Level Study II (SCFLS II) (Naval Surface Warfare Center Dahlgren, 1998). The study determines the optimal mix of Aegis cruisers (CG-52), Aegis guided missile destroyers (DDG-51), and the new destroyers (DD-21) to satisfy national objectives from 2010-2020. Total warfighting surface combatant force requirements will

be derived by an examination of the missions combatants would be tasked in a Major Theater War originating in the Far East about the year 2015.

APL is conducting the study in four phases. First, analysts from APL have met with N-86 and the Office of Naval Intelligence (ONI) to create a scenario that will be realistic, conform to U.S. national objectives, and challenge the flexibility of naval forces. APL created a joint wargame to identify the missions that would be conducted by surface combatants for the year 2015 (Naval Surface Warfare Center Dahlgren, 1999a). Based on the surface combatant tasks identified in the joint wargame, an analysis will be conducted to determine the sufficient numbers and types of ships required for each mission. Finally, the total force requirement will be identified based on the sufficiency analysis, transit time for ships to arrive in theater, and assumed allied ship contributions.

In late April, 1999, representatives from the Chief of Naval Operations, Naval Surface Warfare Center, Dahlgren Division (NSWCDD), Center for Naval Analysis (CNA) and senior representatives from OPNAV conducted a joint wargame for the Far East Major Theater War (Naval Surface Warfare Center Dahlgren, 1999b). They have identified the missions that surface combatants will be assigned. For instance, the joint wargame determined by group consensus the approximate number of ships required for Naval Surface Fire Support (NSFS) during the last week of the war. Applied Physics Laboratory is using the results of this seminar to conduct a sufficiency analysis and determine final force requirements. The completion date for Surface Combatant Force Level Study II is December 1999.

## 2. Mission Descriptions

The wargame has identified five primary missions for surface combatants in the future (Naval Surface Warfare Center, 1999b). Figure 2 displays two of the five missions defined in Table 1.

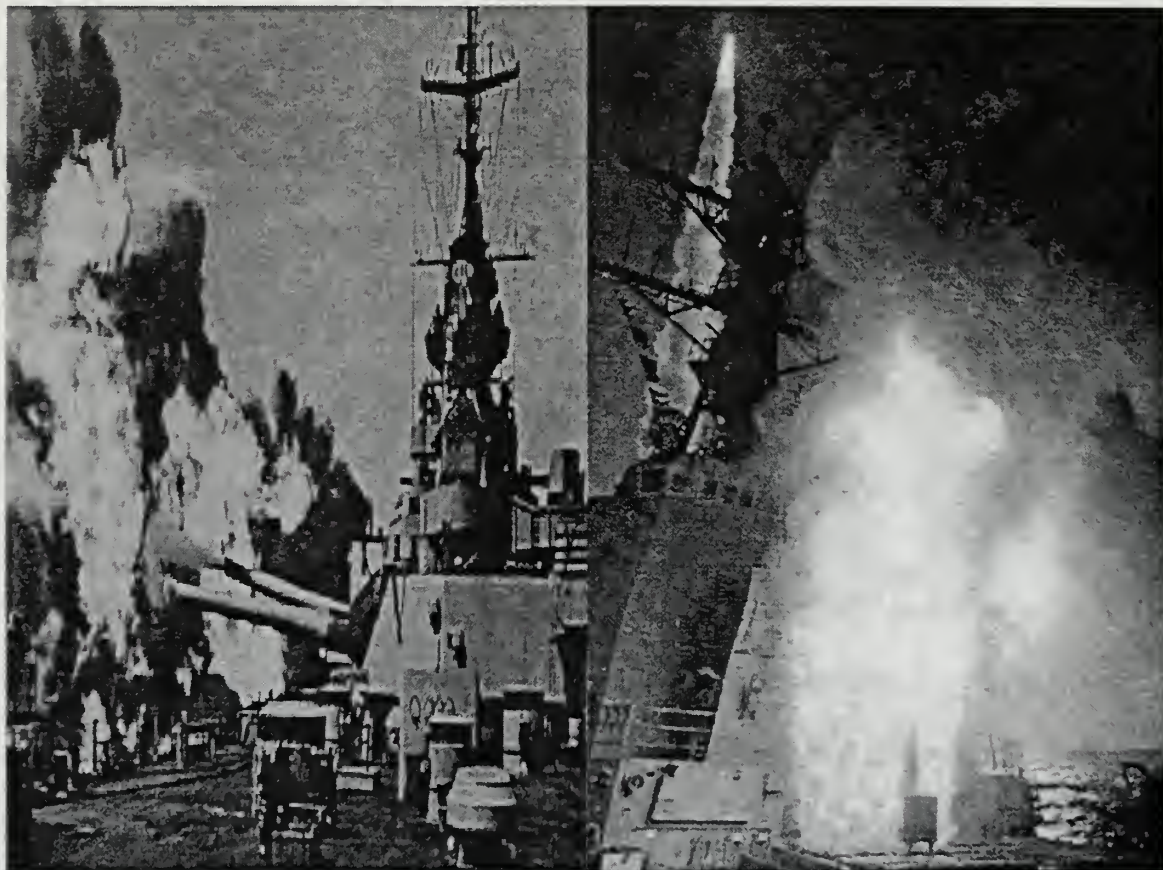


**Figure 2.** Surface Combatant Missions. Surface ships in 2015 will perform missions ranging from classical Naval Surface Fire Support (NSFS) shown on the left to entirely new Theater Ballistic Missile Defense (TBMD), on the right. Present-day missions Theater Air Defense (TAD), Undersea Warfare (USW), and Escort will remain important in the future.

| <b>MISSION</b>                                  | <b>PURPOSE</b>                                                                                                          |
|-------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------|
| <b>Naval Surface Fire Support (NSFS)</b>        | Enables freedom of maneuver by joint and combined ground forces and successful prosecution of the joint land battle     |
| <b>Theater Ballistic Missile Defense (TBMD)</b> | Permits forward-deployed U.S. and coalition forces to operate effectively despite a Theater Ballistic Missile threat    |
| <b>Theater Air Defense (TAD)</b>                | Allows U.S. and coalition forces freedom of action in the littoral by denying enemy exploitation of the air battlespace |
| <b>Undersea Warfare (USW)</b>                   | Denies enemy use of submarines, torpedoes, and mines                                                                    |
| <b>Escort (ESCORT)</b>                          | Enables amphibious and logistics forces to operate in the theater                                                       |

**Table 1.** Description of Missions. These are the five missions for the Far-East Major Theater War scenario.

### 3. Wargame Scenario

The Major Theater War (MTW) scenario for Surface Combatant Force Level Study II occurs in 2015 as the Korean Peninsula is attacked by another power (Naval Surface Warfare Center Dahlgren, 1999c). The conflict lasts approximately 100 days. It is separated into four distinct phases: DETER, DEFEND, BUILDUP, and COUNTER-OFFENSIVE. These phases are defined in Table 2. Minimum and maximum ship requirements are established for each mission per phase. In order to fight the war, all minimum requirements must be satisfied. Conversely, once maximum ship requirements are met, additional ships will not enhance mission accomplishment. The number of ships required per mission is based on the most capable ship class for the mission and assumes 100 percent ship availability. For example, in the build-up phase, a minimum of five CG's is necessary for the entire phase. However, more than eight CG's will not enhance

mission effectiveness. The assignment of less capable classes of ships and replenishment requirements will reduce mission effectiveness. These degradations will be addressed in Chapter II.

| PHASES                  | DESCRIPTION                                                                                                   |
|-------------------------|---------------------------------------------------------------------------------------------------------------|
| <b>DETER</b>            | U.S. and coalition forces attempt to deter enemy from attacking                                               |
| <b>DEFEND</b>           | U.S. and coalition forces slow the enemy ground advance                                                       |
| <b>BUILDUP</b>          | U.S. and coalition ground forces arrive in theater while naval and air forces establish battlespace dominance |
| <b>COUNTEROFFENSIVE</b> | U.S. and coalition forces retake areas occupied by enemy ground forces                                        |

**Table 2.** Phases of the Far East Major Theater War.

## **B. THESIS OUTLINE**

Chapter II defines Fleet Measure of Effectiveness (FMOE) and explains the elements that contribute to its calculation. Chapter III reviews relevant literature, discusses the methodology by which FMOE is calculated, and explains why a distribution for FMOE is preferable to a point estimate. Chapter IV applies this methodology to several case studies. Finally, Chapter V discusses conclusions and recommendations resulting from this study.

## II. FLEET MEASURE OF EFFECTIVENESS

Fleet Measure of Effectiveness (FMOE) is defined as the probability that a fleet will win the war. For instance, a FMOE of .5 indicates that the given mix of surface combatants will have a 50 percent chance of defeating the enemy. Equation (1) defines the components of FMOE.

$$\text{FMOE} = \sum_{m, p} \text{PHASE WEIGHT}_p * \text{MISSION IMPORTANCE}_{m, p} * \text{MISSION EFFECTIVENESS}_{m, p} \quad (1)$$

where m indexes the missions (TBMD, NSFS, TAD, ESCORT, USW)(Table 1), p indexes the phases of the war (DETER, DEFEND, BUILD-UP, and COUNTER-OFFENSIVE) (Table 2), and the remaining terms are defined in this section.

### A. PHASE WEIGHTS

Herein, the weight of each phase is proportional to the length of the phase. For example, the weight of phase DEFEND equals .20 (length of phase, 20 days, divided by length of war, 100 days).

### B. MISSION IMPORTANCE

Surface combatants in this future MTW will perform five primary missions. However, these missions are not of equal importance. Individual mission importance also varies by phase of the war. For instance, decision-makers all stressed during the joint wargame that Naval Surface Fire Support (NSFS) is a much more critical mission than Undersea Warfare (USW) in phase COUNTER-OFFENSIVE. Expert opinion is needed to assess the importance of each mission for all phases of the campaign. The

remainder of this section shows how to use decision-maker input to quantify the importance of these five missions for each phase.

## **1. The Analytic Hierarchy Process**

The Analytic Hierarchy Process (AHP) (Saaty, 1990) is a decision-making tool that can be used to quantify the importance of each mission. For each phase of the war, decision-makers select the more important mission from each pair of missions. They also specify the magnitude of the difference of each pair of missions by assigning a ratio that ranges from one (equally important) to nine (extreme difference). This allows the decision-makers to express the strength of their opinions. The pairwise comparisons are then used to derive a set of weights that quantify the relative importance of each mission (mission weights). See Saaty, 1990 for a complete description of the AHP process.

## **2. Mission Importance Survey**

Expert military judgment is critical to estimate the importance of these missions. A mission importance survey form (Appendix A) is used to solicit these pairwise comparisons from the senior military leaders who participated in the joint wargame (Naval Surface Warfare Center, 1999b). The survey participants are from all four branches of the military and the United States Coast Guard. All fifteen respondents are field grade officers (O5 and above) who have served tours in operational billets. Most have extensive experience with wargaming and strategic planning.

For every phase, an AHP weight is determined for each mission. The AHP weights for each decision-maker sum to one for the missions in each phase. The final product for each phase is fifteen separate sets of weights assessing the importance of

these missions. Each set of weights corresponds to the opinion of each survey participant. When faced with AHP weights from several sources, a common approach (e.g., Saaty, 1990) is to combine them by averaging. In this application, averaging would lose important differences in expert opinion. To capture these differences, we treat the 15 sets of weights as samples from a large population, and estimate probability distributions for mission importance by phase of war. The next chapter will discuss how these distributions of mission weights are estimated. Draws from these distributions give the mission weights used in the calculation of FMOE, as given in Equation (1). Chapter II explains how the variability in expert opinion and hence mission weights drives the variability of Fleet Measure of Effectiveness (FMOE) for a fixed mix of surface combatants.

## **C. MISSION EFFECTIVENESS**

Mission effectiveness is a function of three things: 1) capability of ships assigned (class capability), 2) logistics requirements for the ships (degradation due to activity necessary for replenishment), and 3) interaction between ships assigned and mission effectiveness (synergism) for the ships performing this mission.

### **1. Class Capability**

Requirements for each mission are in terms of ideal ships. An ideal ship belongs to the ship class most capable for that mission. For instance, cruisers (CG-52) are the ideal ship class for Theater Ballistic Missile Defense (TBMD). However, with a fixed fleet size, less than ideal ships may be required to perform missions. In particular, although the DDG class can hold only 91 missiles compared to a maximum of 122 for the

CG class, it still can perform TBMD. The TBMD mission capability of a DDG relative to a CG must be quantified.

Using the conclusions of the wargame, the degradation in mission performance by using the less capable ship class can be estimated. For example, the class capability weight for a DDG performing Naval Surface Fire Support (NSFS) is .5. In other words, two DDG's are required to equal the effectiveness of one destroyer (DD) performing NSFS.

## **2. Replenishment Degradation**

### ***a. Definition***

Surface combatants can not remain on station continuously during the MTW. They must receive food, fuel, and ammunition periodically. Replenishment denotes the resupply of ships either at sea or in port.

Inport replenishment requires a ship leave station and return to port for supplies. Conversely, at-sea replenishment allows the ship to remain underway and receive goods alongside from a supply ship (Figure 3), albeit at some distance from station.



**Figure 3.** A CG (USS Philippine Sea) Receives Fuel From a Fleet Oiler (AO). Surface combatants participating in replenishment are unable to simultaneously perform their primary mission, and the replenishment may take place at some distance from the combatant's mission station.

To conduct at-sea replenishment, the combatant ship (CG, DDG, and DD) leaves station to rendezvous with the supply ship; or the supply ship travels to replenish the combatant on station, a possibly dangerous tactic.

The need to periodically replenish reduces the on-station time for the surface combatant. For instance, if a CG must go off-station for 12 hours to receive fuel from an oiler (Figure 3), the ship may be unable to perform its primary mission. It is inconvenient to simultaneously transfer fuel or ammunition and fire missiles. Therefore, a ship replenishing cannot perform its primary mission.

***b. Ship Endurance and Replenishment Times***

The time a ship can perform an assigned mission without replenishment defines its endurance. A ship always expends fuel and food. In a Major Theater War (MTW), a ship also expends ammunition and missiles at a rapid rate. When a ship runs

out of food, fuel, or munitions, it must replenish. Due to the high tempo of combat operations in this scenario, the ships will run out of munitions before food or fuel.

Replenishment time estimates the time a ship is off station for resupply. The quantities and types of commodities desired by the surface ship will determine replenishment time. Most importantly, ships in this Major Theater War will fire hundreds of missiles. These missiles are placed into Vertical Launch canisters on the ships. Unfortunately, these canisters cannot be loaded at sea. For this scenario, the nearest port safe for loading is in Japan. This will require surface ships performing missions near the Korean peninsula to sail for at least one day just to reach the port and another day to return to their station. Reloading a ship with missiles can be done in a minimum of 12 hours. The ships can minimize off-station time by receiving fuel and food either inport during missile loading or during their transit to and from the port. **It will still take a minimum of three days to replenish ships performing TBMD and TAD with food, fuel, and missiles.** Figure 4 illustrates the replenishment procedure for ships performing Theater Ballistic Missile Defense and Theater Air Defense.

Conversely, ships performing NSFS can replenish ammunition at sea in approximately 12 hours. However, their land attack missiles must be replenished in port. Based on this limitation and the scarcity of land attack missiles, the ships will replenish ammunition frequently. Land attack missile resupply will be rare and may not occur during the war (Lindemann, 1999).

Although the CG class has 122 missile cells compared to 91 for the DDG class, the difference in reload time is only a few hours. The two-day time off-station

required for transit dominates the difference in reload time between classes: class differences for ships are only a minor factor for computing replenishment times.

Due to heavy expenditure of missiles and ammunition during the MTW, replenishment times are a generally a function of the time off-station to receive ammunition and missiles. Using expert judgments from the wargame, replenishment times for each mission and geographic location have been estimated.

*c.      Replenishment Factor*

The replenishment factor is the fraction of time a ship is available to perform its mission if replenishment requirements are considered: ship endurance divided by the sum of ship endurance and replenishment time. The lower the replenishment factor, the more often a ship has to replenish. For instance, a replenishment factor of .80 indicates that ship spends 20 percent of its time off-station in support of replenishment. Table 3 details the replenishment factor for TBMD by location.

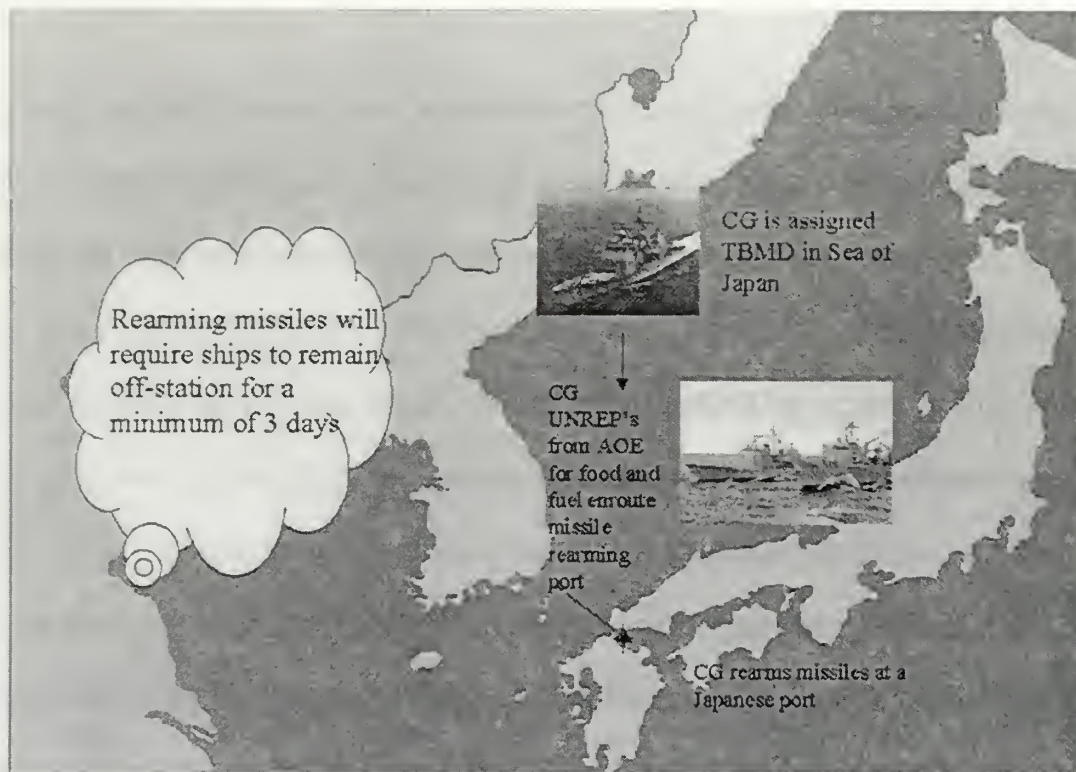


**Figure 4.** Logistics Concept of Operations for Missile Replenishment. Ships that require missile rearming must travel from their assigned station to a port in Japan for reloading. This will take an average of 30 hours to transit from assigned station to the port facility. In addition, rearming missiles inport will require another 12 hours. Although the ship can receive food and fuel from a replenishment ship during transit from the theater to the port facility, a ship will be off-station for approximately 72 hours.

| <b>MISSION-<br/>LOCATION</b>                                                   | <b>ENDURANCE<br/>(days)</b> | <b>REPLENISHMENT<br/>TIME<br/>(days)</b> | <b>REPLENISHMENT<br/>FACTOR</b> |
|--------------------------------------------------------------------------------|-----------------------------|------------------------------------------|---------------------------------|
| <b>THEATER<br/>BALLISTIC<br/>MISSILE DEFENSE<br/>Yellow Sea</b>                | 10                          | 4                                        | .71                             |
| <b>THEATER<br/>BALLISTIC<br/>MISSILE DEFENSE<br/>Eastern Sea of Japan</b>      | 10                          | 3                                        | .77                             |
| <b>THEATER<br/>BALLISTIC<br/>MISSILE DEFENSE<br/>Northern Sea of<br/>Japan</b> | 10                          | 3.5                                      | .74                             |

**Table 3.** Replenishment Factors for Mission Theater Ballistic Missile Defense by location. Ships operating in the Yellow Sea are farther from ports in Japan and are thus off-station longer for replenishment. The mission areas closer to Japanese ports have higher replenishment factors.

### 3. Effective Ships

When a ship is assigned a mission, its performance is degraded by the replenishment factor. Class capabilities less than one also can reduce mission effectiveness. An effective ship represents the residual military value of a ship when replenishment and class capability are considered. Figure 5 illustrates this concept.

A DDG is assigned TBMD in Sea of Japan.



Class Capability

Replenishment factor

Results

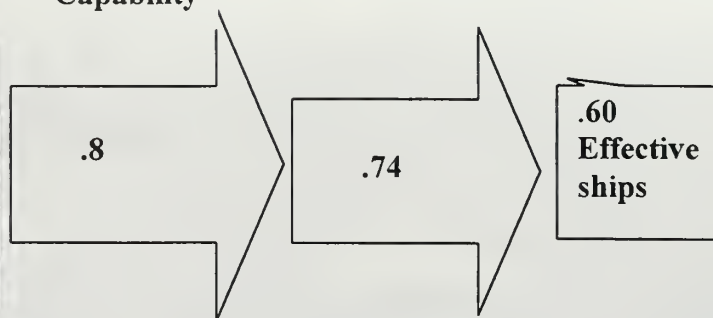


**Figure 5.** Description of an Effective Ship. A guided missile destroyer (DDG) is assigned to perform Theater Ballistic Missile Defense (TBMD). Its class capability is .8 relative to the ideal ship class (cruiser) required for TBMD. A ship assigned TBMD in the Northern Sea of Japan can only remain on-station 74 percent of the time due to replenishment requirements. Therefore, a DDG assigned to TBMD is equivalent to .60 cruiser (CG) performing TBMD continuously.

#### 4. Relationship Between Effective Ships and Mission Effectiveness

The likelihood that each mission can be accomplished is based on the number of effective ships assigned, but it is not a linear relationship. Ships perform missions with synergistic effects. Figure 6 motivates this concept. For example, mission effectiveness for Theater Ballistic Missile Defense (TBMD) in phase DEFEND is zero until a minimum of six effective ships is assigned. Once six effective ships are assigned, effectiveness increases to fifty percent. The rate of increase of mission effectiveness per ship declines after minimum requirements are met. Nevertheless, total mission effectiveness continues to increase and reaches 100 percent when fifteen or more ships that are effective are assigned. Figure 7 illustrates the S-shaped curve for mission effectiveness for TBMD in phase DEFEND.

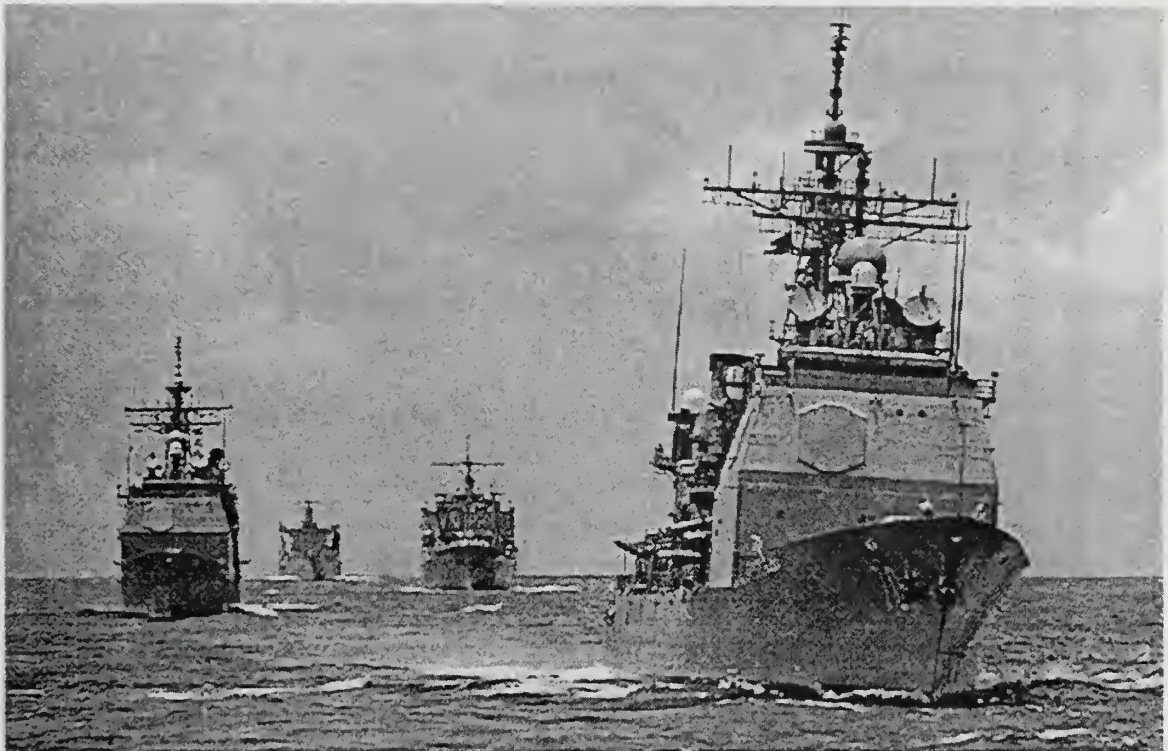


**Figure 6.** Surface Combatants Perform Missions Collaboratively and Synergistically. The intense missions assigned to surface combatants in the future will require close coordination between surface, subsurface, and air units. Most significantly, one ship operating alone cannot achieve mission success. Ships will be required to work in concert to achieve missions. Multiple ships will be necessary and mission effectiveness is dependent on the number of effective ships assigned.

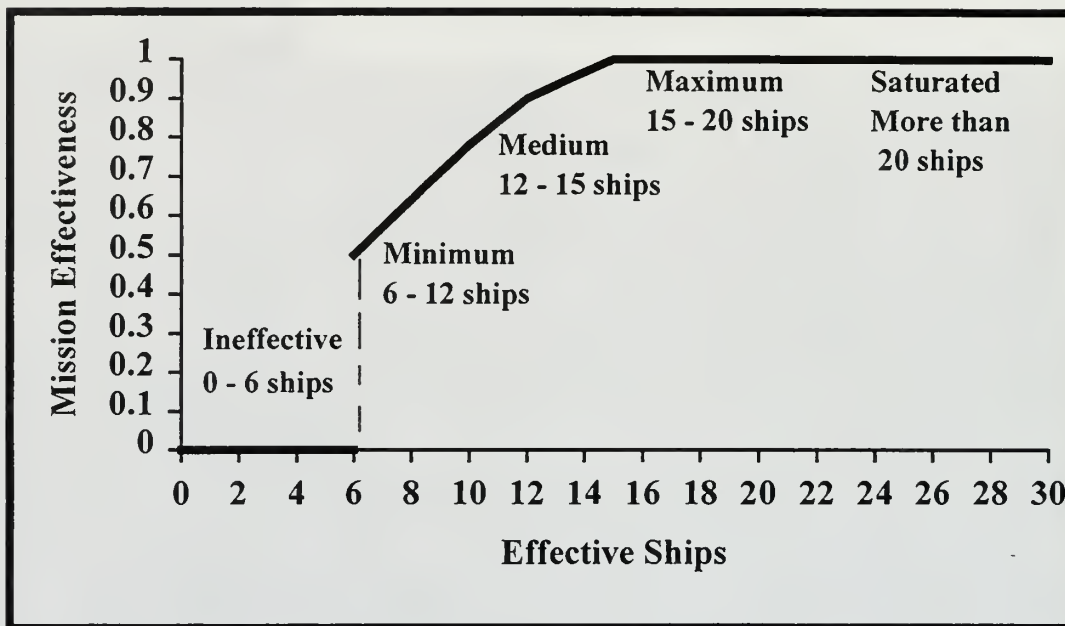


**Figure 7.** Mission Effectiveness as A Function of Effective Ships Assigned. This graph demonstrates the S curve relationship between mission effectiveness and effective ships assigned for Theater Ballistic Missile Defense in the DEFEND phase. At least 6 effective ships are required to achieve minimum requirements. Mission effectiveness increases rapidly as additional ships are added and reaches .9 once 12 effective ships are assigned. Additional ships only marginally improve effectiveness. No benefit is attained by adding ships at the SATURATED level.

### III. LITERATURE REVIEW AND METHODOLOGY

Fleet Measure of Effectiveness (FMOE) is the probability of winning the war. It is a function of several components including phase weight, mission importance, and the effectiveness of a given set of ship assignments to each of the missions. Human judgment is critical in the assessment of the importance of each mission to the eventual outcome of the war. Due to the sometimes-conflicting opinions of multiple decision-makers, the synthesis of expert opinion into a likely distribution for the importance of each mission for each phase is more informative than using an average value. Figure 8 presents a histogram of the importance of Theater Ballistic Missile Defense (TBMD) in the phase COUNTER-OFFENSIVE based on the results of the Analytic Hierarchy Process (AHP) survey (Appendix A) conducted during the joint wargame (Naval Surface Warfare Center Dahlgren, 1999b).

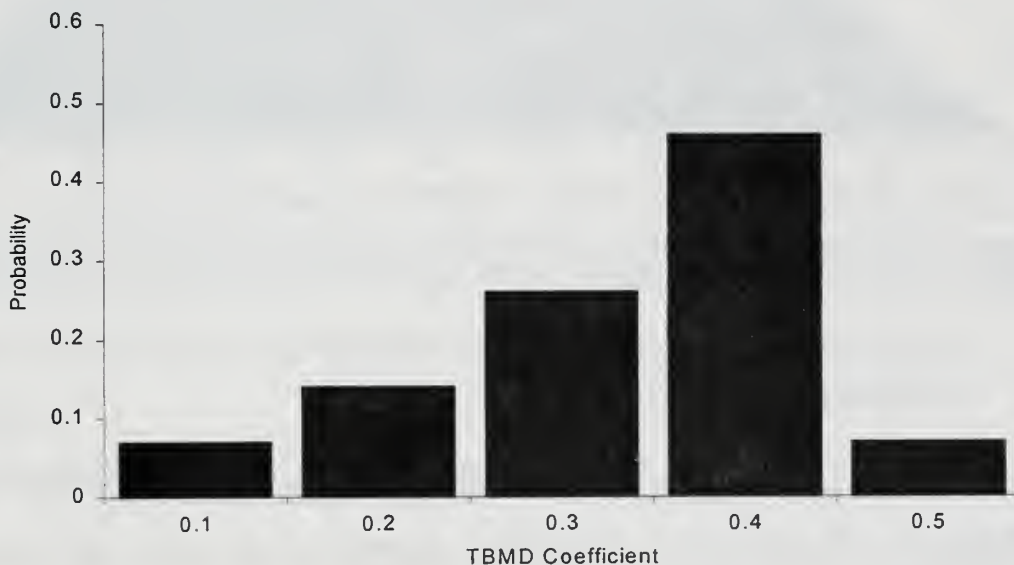


**Figure 8.** The Histogram of Theater Ballistic Missile Defense Weights in the Phase COUNTER-OFFENSIVE.

By treating mission importance weights as random, FMOE for a given fleet of ships also becomes random with a distribution. Figure 9 illustrates such a possible distribution. This distribution reflects the variability in expert opinion and the uncertainty in predicting the probability of winning a future war.

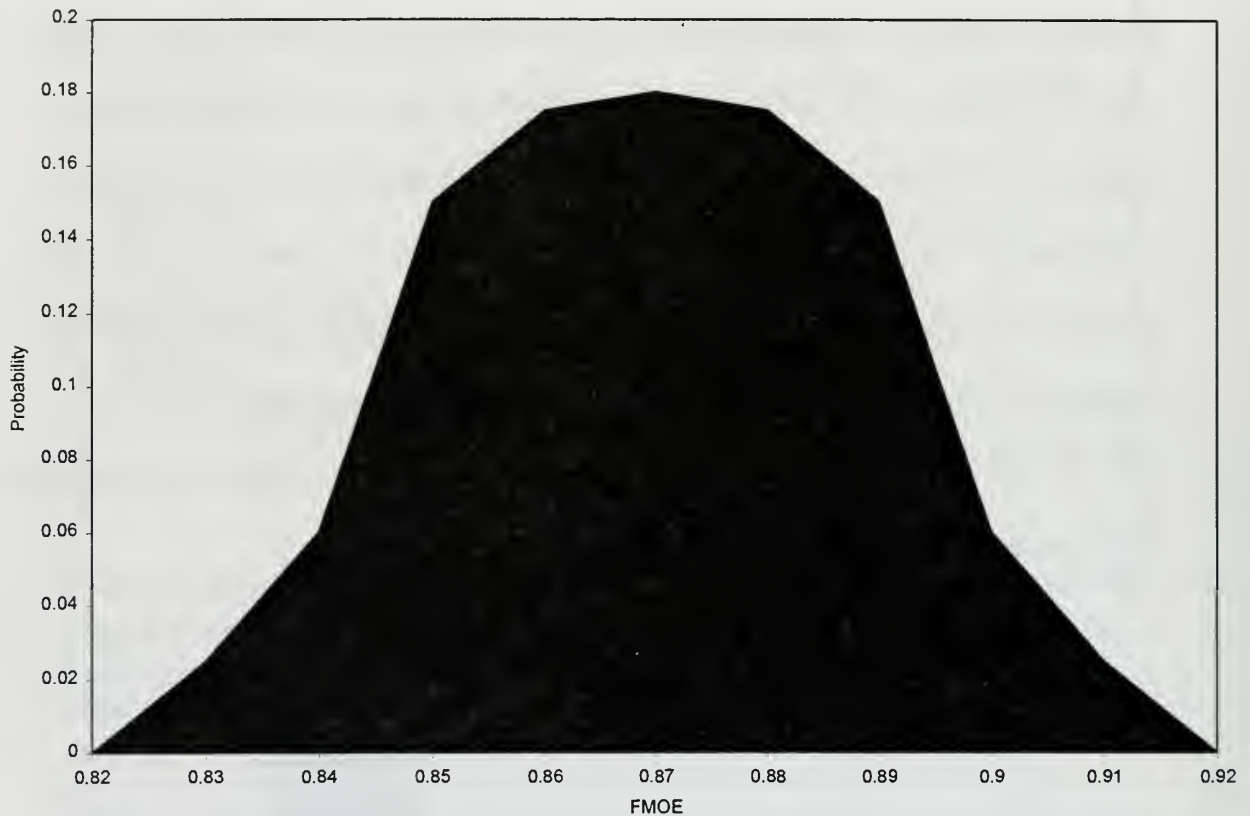


**Figure 9.** The Notional Distribution of FMOE for a Fleet of 17 CG's, 37 DDG's, and 12 DD's

## A. LITERATURE DISCUSSION

This analysis draws on three ideas that have already appeared in the published literature. The first idea is to use distributions instead of average values when combining individual judgments into a group model. As discussed in Chapter II, Analytic Hierarchy Process (AHP) can be used to quantify the importance of missions in each phase. Saaty

(1982 and 1990) recommends geometric means to combine the priorities of multiple judges. For this scenario, we would calculate geometric mean values for the mission weights from the 15 decision-makers during all phases of the conflict. However, mean values do not capture the conflicting judgments of decision-makers. Hauser and Todikmalla (1996) state that point estimates are inappropriate for scenarios where the decision-makers cannot reach consensus.

In addition, the importance of missions in a war is difficult to quantify with a point estimate. A mean value is inappropriate for this scenario because it is impossible to predict exactly how the war will progress. However, a distribution based on the input of decision-makers ensures that FMOE will capture some of this variability. By sampling multiple times for the mission importance weights, a distribution of FMOE based on optimal use of a given fleet is created. This procedure will capture the uncertainty associated with large-scale combat. For example, Basak (1998) shows that AHP distributions provide a stochastic insight into decision-maker preferences.

The next concept is the integration of expert judgments into an optimization model. For example, Korhonen and Wallenius (1990) use this approach to calculate an optimal marketing strategy for a Finnish software company. Saaty (1982) also combines expert judgments and uses linear programming to maximize energy allocation to industries.

The final idea is to integrate simulation with optimization when some of the underlying parameters of the optimization are stochastic. Optimization combined with limited simulation has been documented in the literature as a method to solve stochastic

problems. For example, Jordan and Graves (1995) optimize manufacturing of automobile parts when the demands are stochastic and Shang and Sueyoshi (1995) select the most efficient manufacturing system using AHP, optimization, and simulation.

## **B. METHODOLOGY**

This analysis develops an estimate of FMOE for a fixed fleet by integrating subjective expert opinion with analytical tools including optimization and simulation.

### **1. Development of Mission Importance Distributions**

For each phase of the war AHP mission weights range between 0 and 1 and sum to 1. These weights differ from phase to phase and among experts. To capture differences in weights between phases and variability among experts, we model the five AHP mission weights  $Y_1, \dots, Y_5$  corresponding to TBMD, NSFS, USW, TAD, and ESCORT, respectively, for each phase as positive random variables that sum to one. A reasonable model for the joint distribution of  $Y_1, \dots, Y_5$  is the Dirichlet distribution (Johnson and Kotz, 1972) with parameters  $\alpha_1, \dots, \alpha_5$ . This distribution is a multivariate generalization of the Beta distribution. It is often used to model non-negative random vectors whose elements sum to one (Basak, 1998) as is the case with the AHP mission weights whose sum in each phase equals one. This distribution is used to summarize the opinions of the fifteen experts surveyed during the wargame.

Equation (2) shows the joint density function for the random variables  $Y_1, \dots, Y_5$  with Dirichlet distribution and parameters  $\alpha_1, \dots, \alpha_5$ .

$$\begin{aligned} f(y_1 \dots y_m) &= \frac{\Gamma(\alpha_1 + \dots + \alpha_5)}{\Gamma(\alpha_1) \dots \Gamma(\alpha_5)} y_1^{\alpha_1} \dots y_5^{\alpha_5}, \\ 0 &\leq y_i \quad i = 1 \dots 5, \\ \sum y_i &= 1 \end{aligned} \quad (2)$$

The marginal distributions for  $Y_j$  are Beta with parameters  $\alpha_j$  and  $\sum_{i=1}^5 \alpha_i - \alpha_j$ ,  $j=1 \dots 5$ . In addition,

$$E[Y_j] = \frac{\alpha_j}{\sum_{i=1}^5 \alpha_i} \quad (3)$$

$$\text{Var}[Y_j] = \frac{E(Y_j)(1 - E(Y_j))}{\sum_{i=1}^5 \alpha_i + 1} \quad (4)$$

In order to check that the Dirichlet distribution is a reasonable model for AHP weights, the marginal, empirical distribution of mission weights for each phase are compared to 15 randomly generated observations from a Beta distribution. The parameters for Beta distributions are estimated using the method of moments (Devore, 1995, p. 265). For example, Figure (10) compares the Beta probability plots of the 15 NSFS weights in phase BUILD-UP to three probability plots derived from random observations of a Beta (1.76, 6.47) distribution. Because plots of the three random samples for the Beta distribution and the plot of the NSFS weights appear interchangeable, it is sensible to model the NSFS weights for this phase by a Beta distribution. Similar plots for the mission weights for each phase also confirm that

marginals can be modeled by a Beta and hence a Dirichlet distribution is a sensible choice for the joint distribution of mission weights.

The next step is to estimate  $\alpha_1, \dots, \alpha_5$  for each phase. An algorithm similar to the one of Johnson and Kotz (1972, p. 231) approximates the maximum likelihood estimators (MLE's) for  $\alpha_1, \dots, \alpha_5$ . The algorithm is given in Appendix B. Table 4 lists the MLE's of  $\alpha_1, \dots, \alpha_5$  for each phase.

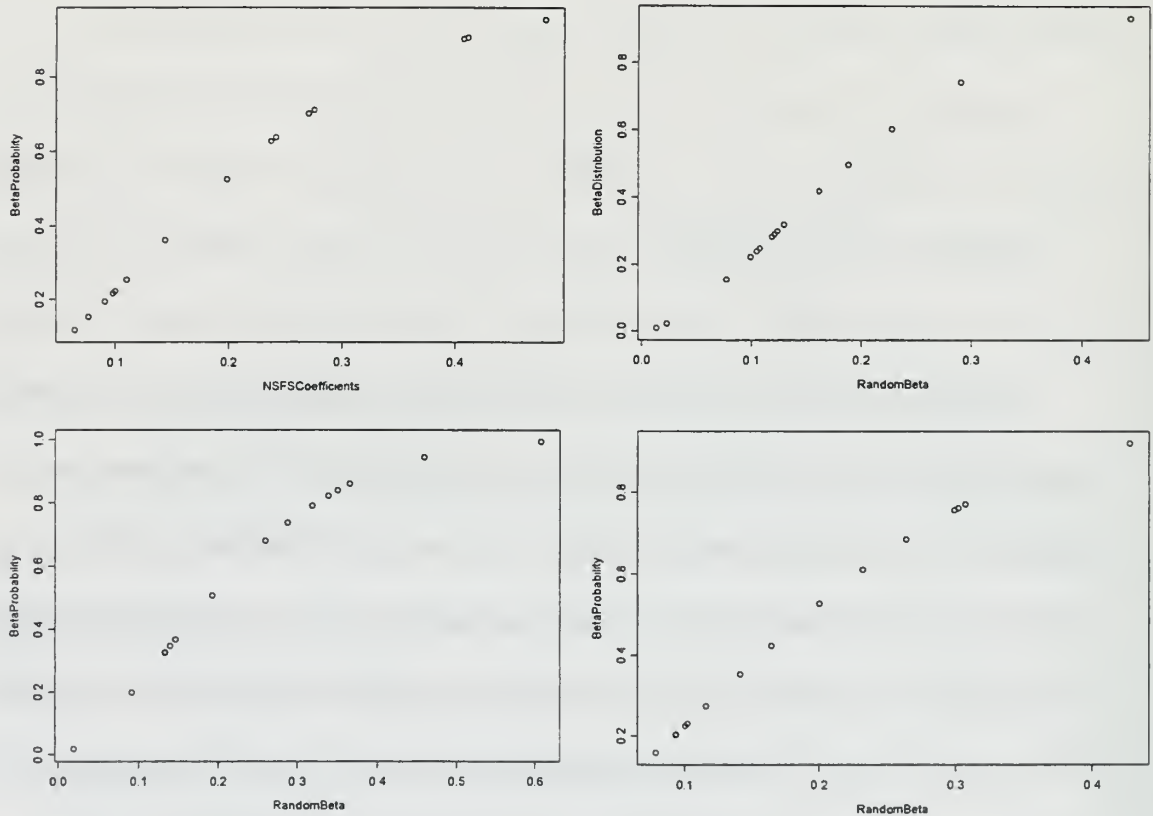


**Figure 10.** Beta Probability Plots. The graph in the upper left corner gives Beta probability plots for the NSFS weights in phase BUILDUP derived from the decision-makers who completed the mission importance survey (Appendix A). For comparison, three additional Beta probability plots are included, each of 15 randomly generated observations from a Beta (1.76, 6.47). Because these plots appear similar, the Beta distribution is a sensible choice to approximate the marginal distribution of AHP weights.

| PHASE                    | TBMD       | NSFS       | USW        | TAD        | ESCORT     |
|--------------------------|------------|------------|------------|------------|------------|
|                          | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ | $\alpha_5$ |
| <b>DETER</b>             | 2.99       | 1.57       | 2.10       | 2.61       | 1.29       |
| <b>DEFEND</b>            | 3.67       | 1.94       | 2.36       | 3.07       | 1.72       |
| <b>BUILDUP</b>           | 4.01       | 2.78       | 2.11       | 2.69       | 1.88       |
| <b>COUNTER-OFFENSIVE</b> | 8.99       | 11.77      | 3.90       | 6.15       | 2.73       |

**Table 4.** Maximum Likelihood Estimators of  $\alpha_1, \dots, \alpha_5$  by phase for all missions in the Major Theater War (MTW).

The MLE's for expected AHP weights are found by substituting the MLE's for the  $\alpha$ 's into Equation (3). These AHP weights are shown in Table 5. From those weights, as the war proceeds, the importance of NSFS increases at the expense of USW and ESCORT. Decision-makers judge that U.S. and coalition forces will have achieved battlespace dominance by phase COUNTER-OFFENSIVE. This will allow American forces to launch extensive NSFS attacks against the enemy.

| PHASE                    | TBMD | NSFS | USW | TAD | ESCORT |
|--------------------------|------|------|-----|-----|--------|
| <b>DETER</b>             | .28  | .15  | .20 | .25 | .12    |
| <b>DEFEND</b>            | .29  | .15  | .18 | .24 | .14    |
| <b>BUILDUP</b>           | .30  | .20  | .16 | .20 | .14    |
| <b>COUNTER-OFFENSIVE</b> | .27  | .35  | .12 | .18 | .08    |

**Table 5.** Maximum Likelihood Estimators of the AHP Weights by phase for all missions in the MTW

The parametric bootstrap (Efron and Tibshirani, 1993) assesses the accuracy of the MLE's. For each of the four Dirichlet distributions, one thousand bootstrap samples of size 15 are generated by sampling from the estimated Dirichlet distribution. For each

bootstrap sample, MLE's are computed for the  $\alpha$ 's. The code for the parametric bootstrap is given in Appendix B. To estimate the bias of the MLE's, Table 6 shows the mean of the  $\alpha$ 's generated by the 1000 bootstrap samples.

| PHASE                    | TBMD<br>$\alpha_1$ | NSFS<br>$\alpha_2$ | USW<br>$\alpha_3$ | TAD<br>$\alpha_4$ | ESCORT<br>$\alpha_5$ |
|--------------------------|--------------------|--------------------|-------------------|-------------------|----------------------|
| <b>DETER</b>             | 3.55               | 1.84               | 2.49              | 3.10              | 1.54                 |
| <b>DEFEND</b>            | 4.25               | 2.23               | 2.74              | 3.54              | 2.01                 |
| <b>BUILDUP</b>           | 4.64               | 3.22               | 2.45              | 3.11              | 2.18                 |
| <b>COUNTER-OFFENSIVE</b> | 10.29              | 13.52              | 4.43              | 7.01              | 3.11                 |

**Table 6.** The Mean  $\alpha$ 's Generated from 1000 Bootstrap Samples. The MLE's generated by the bootstrap methods are approximately 25 percent higher than the MLE's approximated by the Johnson and Kotz algorithm (Table 5).

Comparing Table 4 and Table 6 reveals that the MLE's are biased and that they tend to overestimate the  $\alpha$ 's. From Equation (4), we see that this causes the variability of the AHP weights to be underestimated.

However, the MLE's for the expected AHP weights are relatively unbiased. The average of the expected AHP weights estimated from the bootstrap samples are given in Table 7. These weights are close to the estimated expected AHP weights from the original data (Table 5). In particular, Table 4 (expected AHP weights) and Table 6 (bootstrap generated AHP weights) appear interchangeable to the casual observer. The standard errors of the MLE's of the expected weights are detailed in Table 8. Notice that standard errors are all less than .032.

| PHASE                    | TBMD | NSFS | USW | TAD | ESCORT |
|--------------------------|------|------|-----|-----|--------|
| <b>DETER</b>             | .28  | .15  | .20 | .25 | .12    |
| <b>DEFEND</b>            | .29  | .15  | .19 | .24 | .13    |
| <b>BUILD-UP</b>          | .30  | .20  | .16 | .20 | .14    |
| <b>COUNTER-OFFENSIVE</b> | .27  | .35  | .12 | .18 | .08    |

**Table 7.** AHP Weights for all missions in the MTW generated from the bootstrap samples. These results are almost identical to the AHP weights generated by the Johnson and Kotz algorithm (Johnson and Kotz, 1972, p. 231).

| PHASE                    | TBMD | NSFS | USW  | TAD  | ESCORT |
|--------------------------|------|------|------|------|--------|
| <b>DETER</b>             | .033 | .023 | .027 | .030 | .022   |
| <b>DEFEND</b>            | .031 | .023 | .026 | .027 | .022   |
| <b>BUILDUP</b>           | .030 | .027 | .023 | .025 | .022   |
| <b>COUNTER-OFFENSIVE</b> | .020 | .021 | .014 | .016 | .012   |

**Table 8.** Standard Errors (SE) for the MLE's of the Expected Weights. This indicates that the MLE's for the expected AHP weights are relatively unbiased.

Although the Dirichlet distribution approximation is based on the input of just 15 decision-makers, the estimates of the expected AHP weights are very accurate. However, the MLE's for the  $\alpha$ 's may underestimate the variability of the AHP weights. The consequence is that the final weights may exhibit less variability than they should. The standard errors for MLE's of the expected weights are small enough to insure that the results of the analysis are reasonable.

## 2. Optimization Model

Although the mission importance weights are stochastic, the optimization model calculates FMOE for a fixed realization from the mission weights distributions. By solving the optimization multiple times, each with a different realization from the mission weights, a distribution of FMOE is developed. The remainder of this section details the mathematical formulation and shows how the S curve for mission effectiveness (Figure 7) is approximated by binary and continuous variables (Figure 11).

### *a. Conceptual Model Description*

The Fleet Effectiveness Model (FEFM) assigns ships to missions to maximize their effectiveness over the entire war.

#### **MAXIMIZE:**

Fleet Measure of Effectiveness

#### **Subject to:**

Each ship can perform only one mission per phase.

Replenishment requirements and class capabilities degrade effectiveness of ships.

Mission effectiveness is S-shaped function of effective ships assigned. (Figure 11)

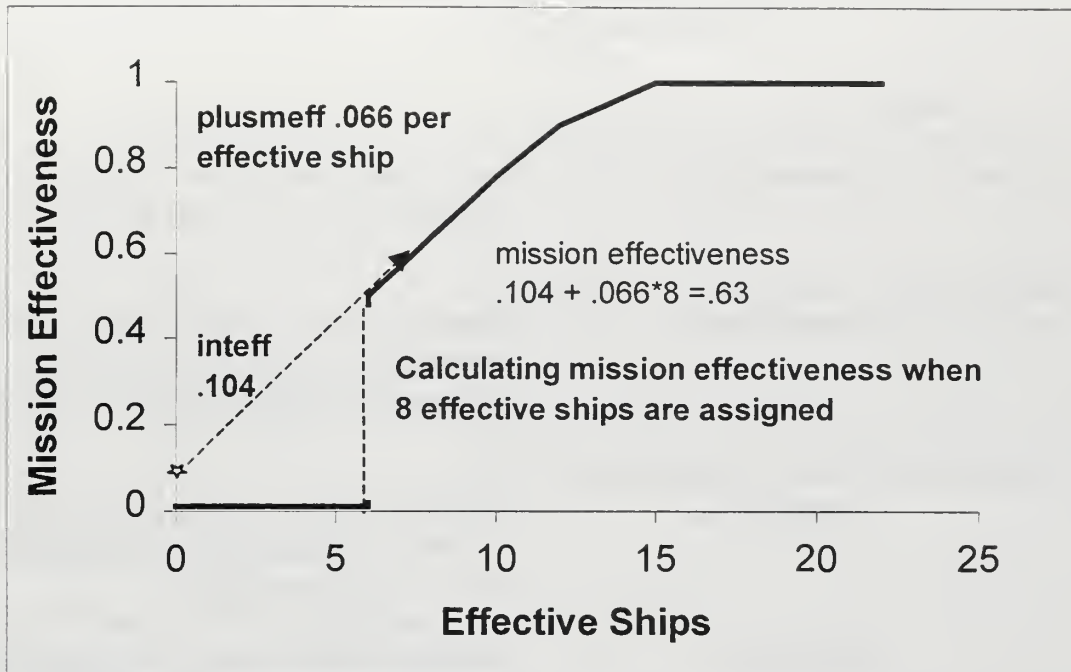


**Figure 11.** Computation of Mission Effectiveness. The piecewise linear function for mission effectiveness (see Figure 7) is calculated using binary and continuous variables. This figure illustrates how mission effectiveness is calculated if 8 effective ships are assigned TBMD in the Eastern Sea of Japan for phase BUILDUP.

#### *b. Mathematical Formulation*

##### Indices:

|                  |                                                                          |
|------------------|--------------------------------------------------------------------------|
| s                | ships (name of ship)                                                     |
| {s} <sub>c</sub> | class of ship (CG, DDG, DD)                                              |
| m                | missions (TBMD, NSFS, TAD, ESCORT, USW)                                  |
| p                | phase (DETER, DEFEND, BUILDUP, COUNTEROFFENSIVE)                         |
| e                | effectiveness level (INEFFECTIVE, MINIMUM, MODERATE, MAXIMUM, SATURATED) |

g geographic area (EAST, WEST, NORTH)  
 Note: missions may be conducted in more than one area e.g.  
 TBMD-NORTH

**Data:**

|                                  |                                                                                                                                                                  |
|----------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| classcap <sub>c,m</sub>          | The capability of a ship in class c to perform mission m (ranges between 0 and 1)                                                                                |
| replenFactor <sub>m,g</sub>      | Percentage of time a ship can perform mission m in geographic region g due to logistics requirements (ranges between .75 and .9)                                 |
| minships <sub>m,e,p,g</sub>      | Minimum number of ships required for mission m at level e during phase p for geographic location g (effective ships)                                             |
| inteff <sub>m,e,p,g</sub>        | The mission effectiveness intercept for mission m at effectiveness level e during phase p at geographic location g (see Figure 11)                               |
| plusmeff <sub>m,e,p,g</sub>      | The increase in mission effectiveness obtained by adding another effective ship at level e for mission m during phase p at geographic location g (see Figure 11) |
| missionImportance <sub>m,p</sub> | Analytic Hierarchy Process based mission weights for mission m at phase p (ranges between 0 and 1)                                                               |
| geoImportance <sub>m,g</sub>     | The weight of each mission m at each geographic location g (ranges between 0 and 1)                                                                              |
| phaseWeight <sub>p</sub>         | The weight of each phase p<br>(Days in each phase divided by days in war)                                                                                        |

**Decision variables**

|                           |                                                                                                                                                                   |
|---------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| ASSIGN <sub>s,m,p,g</sub> | = 1 if ship s is assigned mission m during phase p for geographic area g. It is understood that not all combinations of subscripts are possible.<br>= 0 otherwise |
|---------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|

|                      |                                                                                                                            |
|----------------------|----------------------------------------------------------------------------------------------------------------------------|
| $MINMET_{m,e,p,g}$   | = 1 if minimum ship requirements for mission m at level e during phase p at geographic location g are met<br>= 0 otherwise |
| $MISSIONEFF_{m,p,g}$ | the mission effectiveness for each mission m and during each phase p at geographic location g                              |
| FMOE                 | fleet measure of effectiveness. The percentage of all weighted missions completed during the war.                          |
| $EShips_{m,e,p,g}$   | number of effective ships for each mission m at level e during phase p at geographic location g                            |

## FORMULATION

### Maximize objective function

$$FMOE = \sum_{m,p,g} mission\ Importance_{m,p} * phaseWeight_p * geoImportance_{m,g} * MISSIONEFF_{m,p,g} \quad (5)$$

FMOE= (1)

Subject to

$$\sum_s \text{classcap}_{s,m} * \text{replenFactor}_{m,g} * \text{ASSIGN}_{s,m,p,g} = \sum_e \text{ESHIPS}_{m,e,p,g} \quad \forall m,p,g \quad (6)$$

$$\sum_e (\text{intmeff}_{m,e,p,g} * \text{MINMET}_{m,e,p,g} + \text{plusmeff}_{m,e,p,g} * \text{ESHIPS}_{m,e,p,g}) = \text{MISSIONEFF}_{m,p,g} \quad \forall m,p,g \quad (7)$$

$$\sum_{m,g} \text{ASSIGN}_{s,m,p,g} = 1 \quad \forall s,p \quad (8)$$

$$\text{minships}_{m,e,p,g} * \text{MINMET}_{m,e,p,g} \leq \text{ESHIPS}_{m,e,p,g} \quad \forall m,e \neq \text{'INEFFECTIVE'},p,g \quad (9)$$

$$\text{minships}_{m,e+1,p,g} * \text{MINMET}_{m,e,p,g} \geq \text{ESHIPS}_{m,e,p,g} \quad \forall m,e \neq \text{'SATURATED'},p,g \quad (10)$$

$$\sum_e \text{MINMET}_{m,e,p,g} = 1 \quad \forall m,p,g \quad (11)$$

$$\text{ASSIGN}_{s,m,p,g} \in \{0,1\} \quad \forall s,m,p,g \quad (12)$$

$$\text{MINMET}_{m,e,p,g} \in \{0,1\} \quad \forall m,e,p,g$$

$$\text{ESHIPS}_{m,e,p,g} \geq 0 \quad \forall m,e,p,g \quad (13)$$

$$\text{MISSIONEFF}_{m,p,g} \geq 0 \quad \forall m,p,g$$

Equation (5) represents the computation of FMOE. Equation (6) calculates effective ships for each mission and phase. Equation (7) computes the effectiveness of each mission for each phase (see Figure 11). Equation (8) ensures that each ship is assigned to only one mission per phase. Equations (9) and (10) ensure that levels of mission effectiveness are attained only if enough effective ships are assigned. Equation (11) ensures only one level of effectiveness per mission and phase. Specifications (12) require binary decisions are made for ship assignment and determination of mission effectiveness levels. Specifications (13) ensure FMOE, effective ships, and mission effectiveness are positive variables.

### 3. Expected Value Analysis

Saaty (1982 and 1990) recommends geometric means to combine the priorities of decision-makers. Figure 12 shows the Fleet Measure of Effectiveness (FMOE) results for a fleet of 17 CG's, 37 DDG's, and 12 DD's using both the Dirichlet distribution and the geometric mean for mission weights. The FMOE point estimate generated from the geometric mean for mission weights is close to the arithmetic mean of the FMOE distribution derived from the Dirichlet distribution of mission weights. However, the FMOE distribution displays more of the uncertainty involved in large-scale combat.

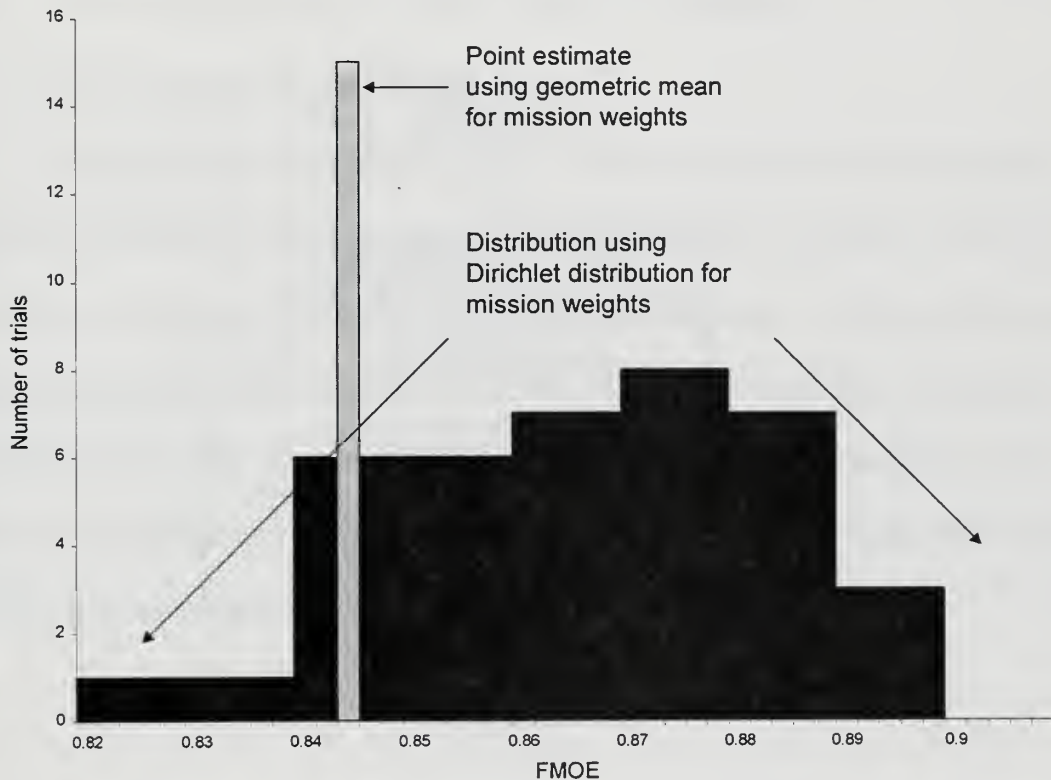


**Figure 12.** A distribution of FMOE expresses the uncertainty associated with measuring fleet effectiveness. A point estimate using the geometric mean for mission weights does not display this variability.

#### 4. Integration of Distributions with the Optimization Model

Figure 13 shows how a distribution of FMOE is calculated. First, a specific fleet mix is designated. A sample of Analytic Hierarchy Process (AHP) mission weights are drawn from the Dirichlet distribution associated with each phase. A total of 20 mission weights (5 missions by 4 phases) is generated for each sample. The optimization model is then run with these weights and solves for FMOE. Another set of 20 mission importance weights is sampled from the Dirichlet distributions until 40 complete samples are drawn. FMOE is calculated for each sample of mission importance weights. The final product is 40-observations of FMOE for a given fleet.

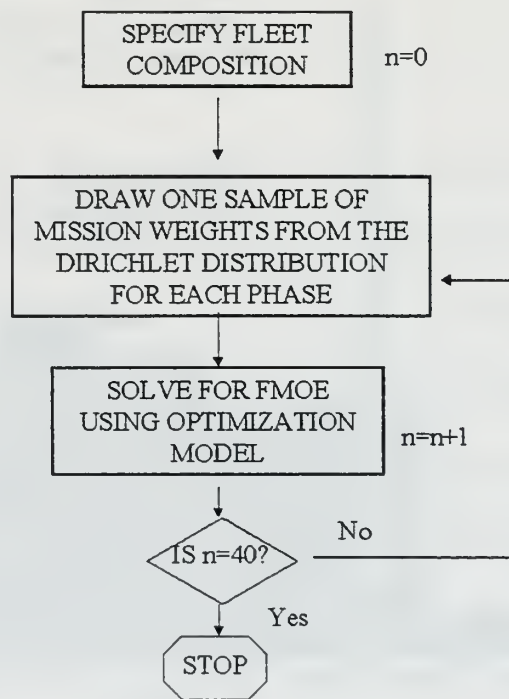


**Figure 13.** This diagram illustrates the process of computing Fleet Measure of Effectiveness (FMOE) 40 times for a fixed fleet of ships.

## IV. CASE STUDIES AND RESULTS

This chapter will show how FEFM can aid in the design of an effective mix of surface combatants. In particular, case studies show how the composition of ships participating in the war (ship mix), logistics support, and the mission capability of each ship class contribute to overall fleet effectiveness.

Fleet Effectiveness Model (FEFM) is implemented in the General Algebraic Modeling System (Brooke, Kendrick, Meeraus, and Raman, 1997). FEFM is solved using CPLEX 6.5 with Gams Version 2.50, Distribution 18.1 (CPLEX Optimization, Inc. 1999). Results are guaranteed to be within 2 percent of optimality.

### A. COMPARISON OF SHIP MIXES

Although FMOE increases with each additional surface combatant, the correct mix of ships from each of the three classes (CG, DDG, and DD) is critical to success in a war. Figure 14 motivates this concept. FMOE for each fleet size is optimized forty times using random sets of mission weights drawn from the Dirichlet distributions for each phase (Figure 13). Each fleet mix uses the same forty sets of mission weights. Both fleets have 66 ships but the mix identified by the Surface Combatant Force Level Study II (SCFLS II) is clearly superior.

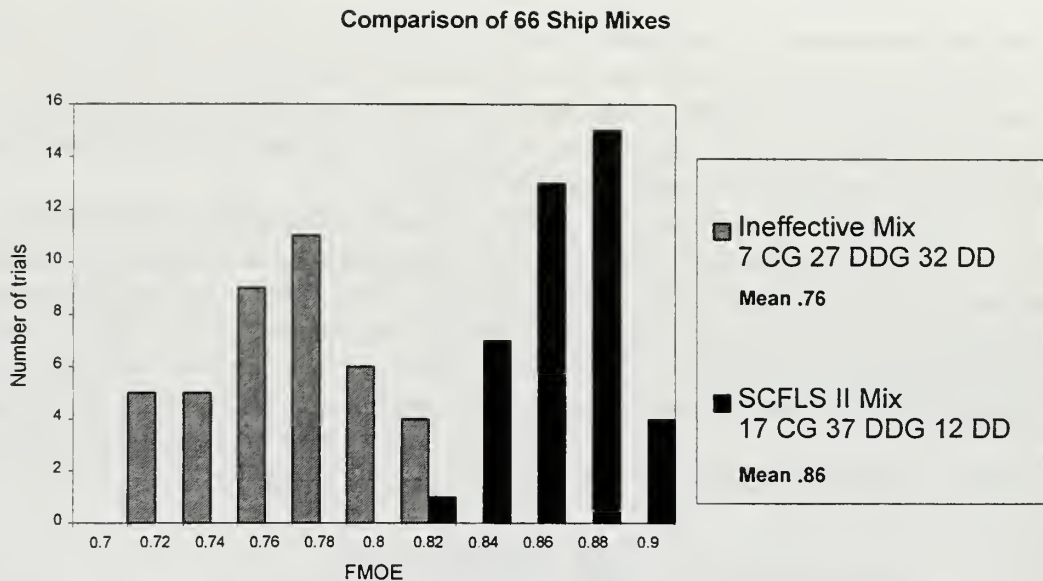


**Figure 14.** Comparison of ship mix distributions. The ship mix (17 CG 37 DDG 12 DD) proposed by the Surface Combatant Force Level Study II (SCFLS II) is superior to most 66-ship mixes. It is clearly superior to the “ineffective” 66-ship mix (7 CG 27 DDG 32 DD).

## B. LOGISTICS IMPACT ON FMOE

The proper number and mix of surface combatants does not guarantee success in a Major Theater War. As discussed in Chapter II, surface combatants performing Theater Ballistic Missile Defense (TBMD) and Theater Air Defense (TAD) must reload missiles in Japanese ports. Ships will be off-station a minimum of three days for missile replenishment. However, if US ships can not reload in Japanese ports, they must travel to Guam for missile replenishment. These ships would be off-station at least seven days. Figure 15 examines the impact that missile replenishment location has on FMOE.

### Impact of Logistics on FMOE

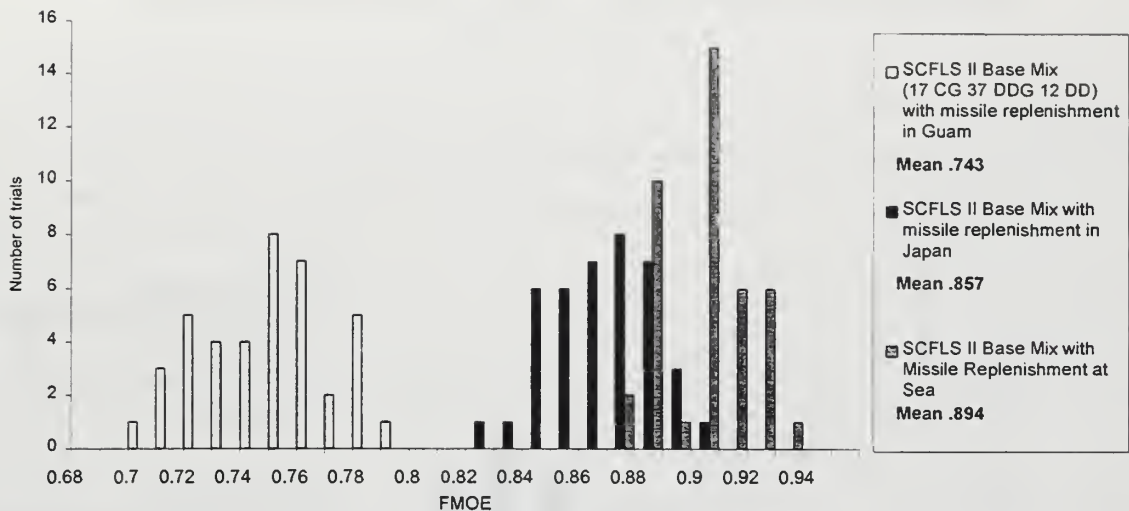


**Figure 15.** Logistics impact on Fleet Effectiveness. While all three fleets consist of 17 CG's, 37 DDG's, and 12 DD's, ships that can replenish in Japan are much more effective than surface combatants that must travel to Guam for missile replenishment. Conversely, if the ability to perform VLS missile replenishment at sea is developed by 2015, FMOE would increase significantly.

This case study corroborates a key lesson the United States learned during the Persian Gulf War: forward logistics bases are required to support missile and ammunition replenishment of naval assets in a MTW (Department of Defense, 1992). More ships would be required to fight an MTW in the Far East if missile replenishment is not available in theater. Nevertheless, Figures 16 and 17 show that even with a significantly larger fleet it is difficult to compensate for the loss of missile replenishment in theater.

More ships are required if in-theater missile replenishment is not possible

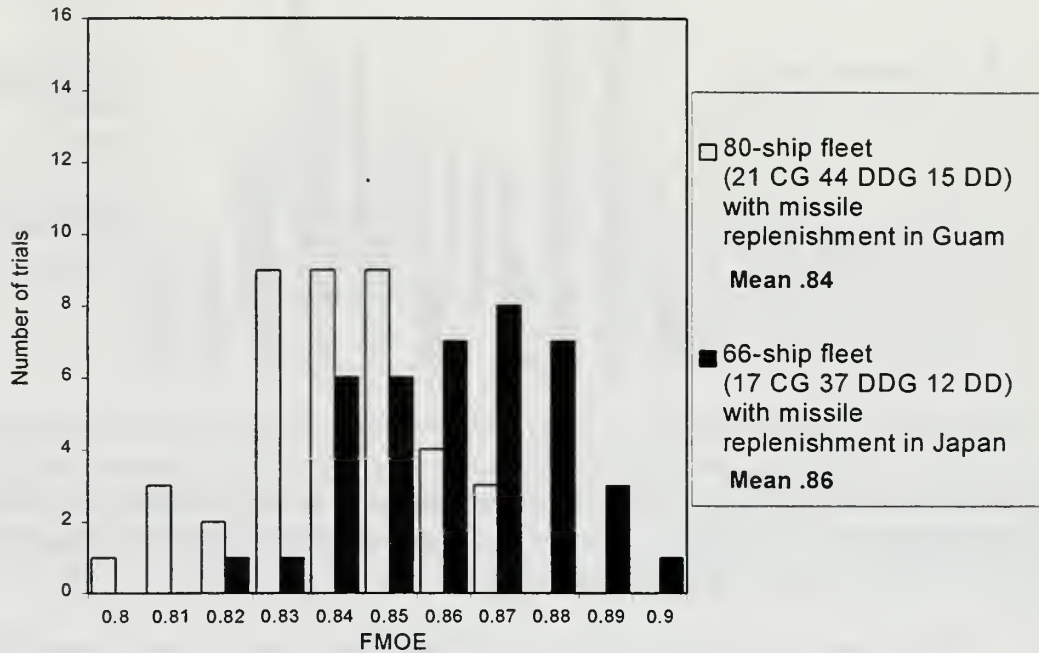


**Figure 16.** Poor logistics support will inhibit an otherwise effective fleet of surface combatants. An 80-ship mix with missile replenishment in Guam is about as effective as a 66-ship fleet that reloads missiles in Japan.

### Interpreting FMOE results

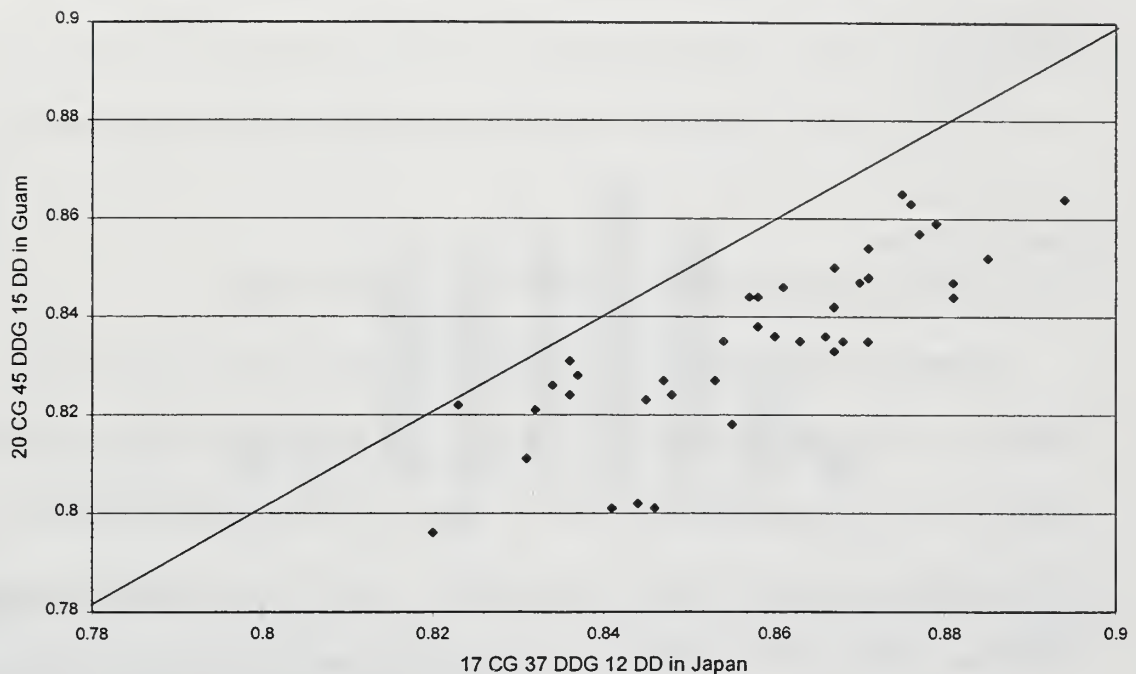


**Figure 17.** Each point in this figure is a paired observation. The mission weights are identical, but the fleet mix and replenishment methods are different. If all points lie beneath the diagonal, the 66-ship fleet (17 CG 37 DDG 12 DD) with missile reload in-theater would be superior. On the other hand, if all the points are above the diagonal, the 80-ship fleet (21 CG 44 DDG 15 DD) that conducts missile replenishment in Guam would be the preferred choice. This is a powerful tool to compare the fleet mixes because it “corrects” for the randomness in the weights.

The 66-ship fleet appears to be more effective than the 80-ship mix that replenishes in Guam. Figures 16 and 17 suggest that the 66-ship fleet with replenishment in Japan is superior. The mean FMOE of the 66-ship mix is 2 percent higher than the 80-ship fleet (Figure 16). In fact, the test statistic value of the paired t

test is 20.35 with a p-value of .0001 (Devore, 1995, p. 368). This strongly suggests that the 66-ship fleet is superior to the 80-ship fleet.

However, even with this evidence it is not correct to conclude that the large fleet (Guam replenishment) does not compensate for the loss of missile replenishment capability in theater. Most importantly, FEFM's representation of the MTW is coarse and is able to effectively discern "large" differences in fleet effectiveness but not "small" ones.

### **C. CLASS CAPABILITY COMPARISONS USING FMOE**

Fleet Measure of Effectiveness (FMOE) distributions can be used to help determine mission capabilities for each ship class. As discussed in Chapter II, the class capability for a DDG performing NSFS is .5. Two DDG 's are needed to match the mission effectiveness of one DD performing NSFS. Figure 18 show that designing the DDG class without NSFS capability would not significantly reduce FMOE for this MTW.

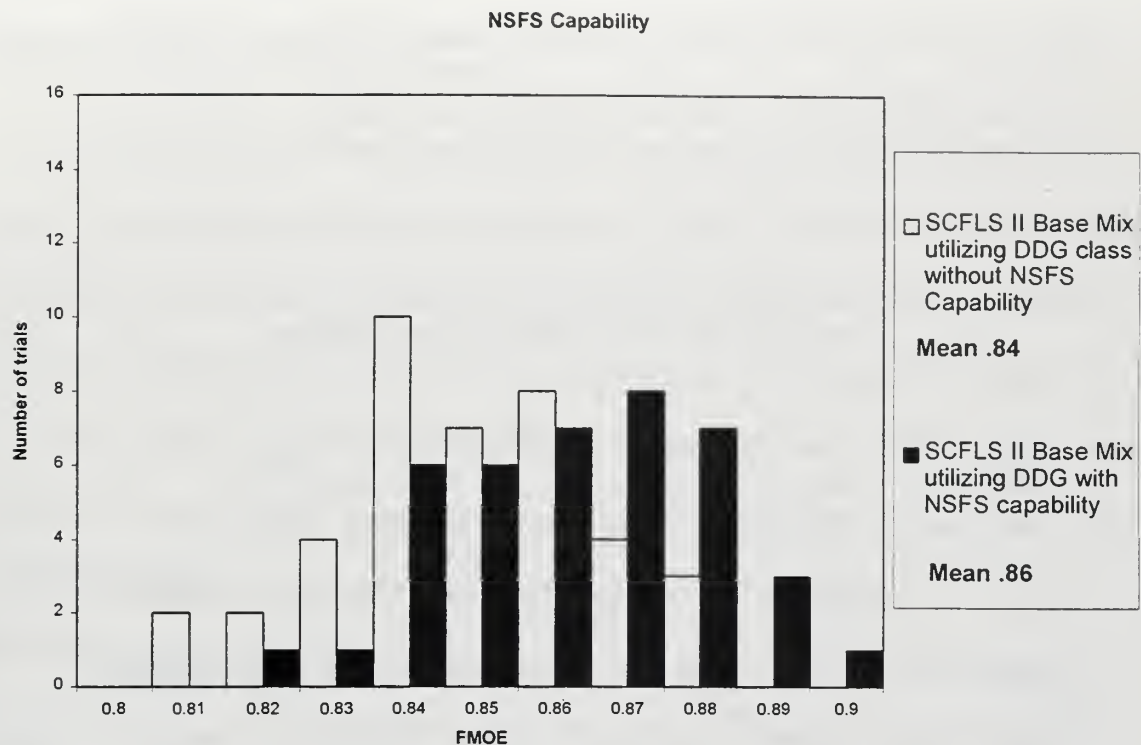


**Figure 18.** NSFS capability options for the DDG class. The 66-ship mix identified by SCFLS II but without NSFS capability for the DDG class is only slightly less effective than the same ship mix with NSFS capability.

#### **D. FAST COMBATANT OPTION**

FMOE distributions also can measure the performance of new classes of ships. In particular, although the DDG class can perform all five missions, the previous section shows that the NSFS capability on the DDG does not enhance overall fleet effectiveness for the Far East Major Theater War. Providing ship classes with multi-mission capability does not always translate into higher fleet effectiveness.

Rear Admiral (Retired) Worthington (1994) proposes building “Fast Combatant” class ships. These ships would conduct Escort and Undersea Warfare near the beach and would allow the DDG class ships to focus on Theater Air Defense and Theater Ballistic

Missile Defense. Worthington argues against assigning a billion-dollar ship to perform a mission that can be accomplished by a smaller, cheaper ship class. Figure 19 uses FEFM to evaluate a fleet of ships with Fast Combatant class ships.

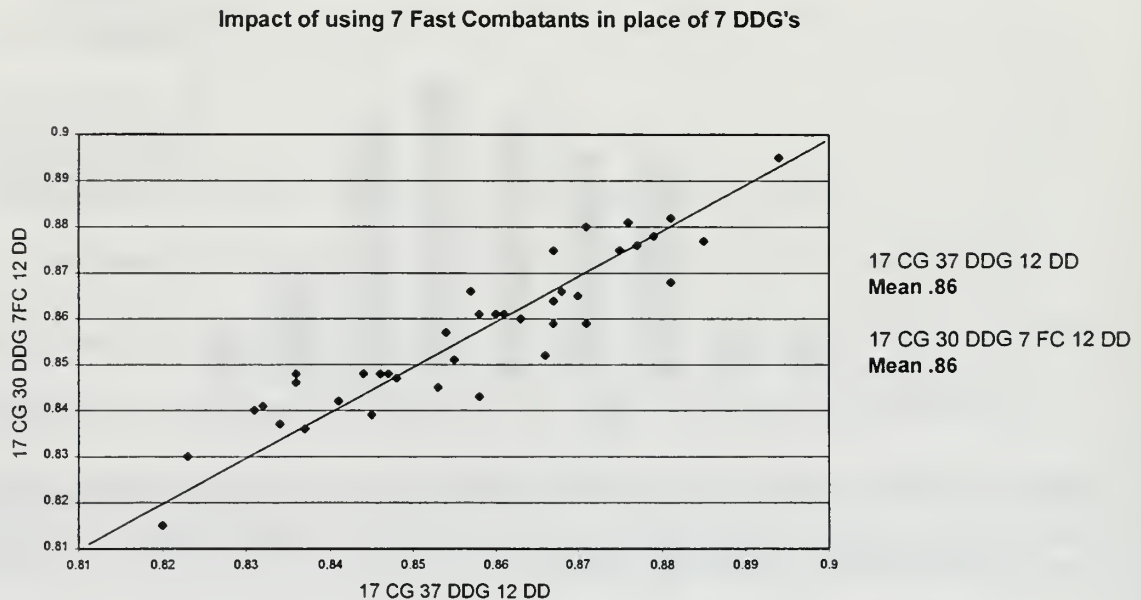


**Figure 19.** Measuring the Effectiveness of the Fast Combatant (FC) Class. The two fleets in this case study are indistinguishable as the data points are almost equally distributed on each side of the center diagonal. Therefore, using 30 DDG's and 7 FC's instead of 37 DDG's causes no discernible loss of effectiveness. Furthermore, each FC class ship would cost only a fraction of the price of a DDG (Worthington, 1994).

## **V. CONCLUSIONS**

### **A. SUMMARY OF RESULTS**

The Fleet Effectiveness Model (FEFM) provides decision-makers with a tool to measure the performance of surface combatants in a Major Theater War. FEFM not only measures the performance of a fleet of ships but also illustrates how surface combatant mission capabilities affect fleet performance. It also verifies the strong relationship between logistics support and fleet effectiveness. Most importantly, it allows planners to address the many “what if” scenarios concerning fleet composition. By analyzing these scenarios, FEFM can assist in the design of future surface combatants.

This thesis measures fleet effectiveness for a future MTW in Asia. Further studies are needed to examine how surface combatants perform in different theaters versus more (or less) capable enemies. Finally, a detailed comparison between cost data and FMOE for different ship mixes may help the Navy efficiently utilize every ship procurement dollar.

### **B. METHODOLOGICAL INNOVATIONS**

FEFM is not a finished product. However, this thesis shows that expert opinion combined with quantitative Operations Research methods can analyze a difficult problem: the collective performance of surface ships in a war. This novel approach employs techniques from decision analysis, statistics, simulation, and optimization. The Army and the Applied Physics Laboratory at The Johns Hopkins University (JHU/APL) are considering a similar methodology to analyze the Army force levels necessary to win smaller-scale conflicts.

Acquiring and designing ships will always be an expensive process. FEFM demonstrates that mathematical programming can help measure fleet effectiveness and therefore aid in the design of a more effective fleet.

## APPENDIX A. MISSION IMPORTANCE SURVEY

**Purpose:** This survey is designed to collect pairwise comparison data for the primary missions in the MRC. The missions will be compared for each of the four phases. You have been briefed on the importance and intent of this survey.

Please use the numerical scale below to describe the relationship between mission.

- |   |                  |
|---|------------------|
| 1 | EQUAL/SAME       |
| 2 | MODERATE         |
| 5 | STRONG           |
| 7 | VERY STRONG      |
| 9 | EXTREMELY STRONG |

2, 4, 6, and 8 are intermediate values.

Finally, please circle either ADVANTAGE or DISADVANTAGE when completing each comparison.

### Example

TBMD has a 3 advantage/disadvantage versus NSFS

**The following questions apply to Phase 1 (DETER)**

TBMD has a \_\_\_\_\_ advantage/disadvantage versus NSFS  
TBMD has a \_\_\_\_\_ advantage/disadvantage versus USW  
TBMD has a \_\_\_\_\_ advantage/disadvantage versus TAD  
TBMD has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

NSFS has a \_\_\_\_\_ advantage/disadvantage versus USW  
NSFS has a \_\_\_\_\_ advantage/disadvantage versus TAD  
NSFS has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

USW has a \_\_\_\_\_ advantage/disadvantage versus TAD  
USW has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

TAD has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

**The following questions apply to Phase 2 (DEFEND)**

TBMD has a \_\_\_\_\_ advantage/disadvantage versus NSFS  
TBMD has a \_\_\_\_\_ advantage/disadvantage versus USW  
TBMD has a \_\_\_\_\_ advantage/disadvantage versus TAD  
TBMD has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

NSFS has a \_\_\_\_\_ advantage/disadvantage versus USW  
NSFS has a \_\_\_\_\_ advantage/disadvantage versus TAD  
NSFS has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

USW has a \_\_\_\_\_ advantage/disadvantage versus TAD  
USW has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

TAD has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

**The following questions apply to Phase 3 (BUILDUP)**

TBMD has a \_\_\_\_\_ advantage/disadvantage versus NSFS  
TBMD has a \_\_\_\_\_ advantage/disadvantage versus USW  
TBMD has a \_\_\_\_\_ advantage/disadvantage versus TAD  
TBMD has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

NSFS has a \_\_\_\_\_ advantage/disadvantage versus USW  
NSFS has a \_\_\_\_\_ advantage/disadvantage versus TAD  
NSFS has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

USW has a \_\_\_\_\_ advantage/disadvantage versus TAD  
USW has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

TAD has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

**The following questions apply to Phase 4 (COUNTER-OFFENSIVE)**

TBMD has a \_\_\_\_\_ advantage/disadvantage versus NSFS  
TBMD has a \_\_\_\_\_ advantage/disadvantage versus USW  
TBMD has a \_\_\_\_\_ advantage/disadvantage versus TAD  
TBMD has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

NSFS has a \_\_\_\_\_ advantage/disadvantage versus USW  
NSFS has a \_\_\_\_\_ advantage/disadvantage versus TAD  
NSFS has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

USW has a \_\_\_\_\_ advantage/disadvantage versus TAD  
USW has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

TAD has a \_\_\_\_\_ advantage/disadvantage versus ESCORT

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## APPENDIX B. S-PLUS FUNCTIONS

This appendix contains S-plus functions [S-plus 4.5, MathSoft Incorporated, 1999] used in the thesis.

```
dirchlet.approx.mle
function(X)
{
#This function estimates the approximate MLE using an approximation
#for the MLE of a Beta (Johnson and Kotz) adapted for the Dirichlet
#distribution
#alpha = a vector which contains the parameters of the Dirichlet
  n<-dim(X)[1]
  k<-dim(X)[2]
  prod.col<-apply(X,2,prod)^(1/n)
  s<-sum(prod.col)
  alpha<-(0.5*(k-1)*prod.col +.5 -.5*s)/(1-s)
  alpha
}
```

```

dirichlet.boot
function(B=100, n=15, alpha)
{
  #n = size of one sample (n=15)
  #B = the number of bootstrap samples of size n (B = a large number)
  #alpha = a vector which contains the parameters of the Dirichlet
  #dirichlet.approx.mle estimates the approximate MLE
  dirchlet.approx.mle
    function(X)
    {
      n<-dim(X)[1]
      k<-dim(X)[2]
      prod.col<-apply(X,2,prod)^(1/n)
      s<-sum(prod.col)
      alpha<-(0.5*(k-1)*prod.col +.5 -.5*s)/(1-s)
      alpha
    }
  k<-length(alpha)
  alpha.boot<-matrix(0, nrow = B, ncol = k)
  for (i in 1:B)
  {
    Y<-matrix(rgamma(k*n, shape = alpha), byrow = T, ncol =
      k)

    S.Y<-matrix(rep(apply(Y,1,sum),k), ncol=k)
    X<-Y/S.Y
    alpha.boot[i,]<-dirichlet.approx.mle(X)
  }
  alpha.boot
}

```

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9. Professor Linda K. Nozick ..... 2  
School of Civil and Environmental Engineering  
Cornell, University  
Ithaca, NY 14853

10. Professor Gerald G. Brown (Code OR/Bw) ..... 2  
Department of Operations Research  
Naval Postgraduate School  
Monterey, CA 93943-5000
11. Professor Lyn Whitaker (Code OR/Wh)..... 1  
Department of Operations Research  
Naval Postgraduate School  
Monterey, CA 93943-5000
12. Professor David Schrady (Code OR/So)..... 1  
Department of Operations Research  
Naval Postgraduate School  
Monterey, CA 93943-5000
13. LT Michael A. Crary..... 2  
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